

ARISTOTLE ON THREE-DIMENSIONALITY

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Abstract.

There have been only a few studies on Aristotle's assertion that a body is perfect only if it contains no more and no less than three dimensions. Three-dimensionality thus is said to satisfy the spatial existence of things. However, one might object that Aristotle did not provide any arguments in favour of his thesis. I shall argue that he did. Admittedly, the proofs he offered in *On Heaven* cannot be taken as self-standing; I will argue Aristotle was well aware of this. I will try to show that they must be related to other metaphysically crucial concepts, such as nature, principle, place and limit. In light of this, I will say that no body is presupposed for us to rightfully conceive the proper nature of place. Finally, I will claim that the dimensionality of things in space is genuinely based upon the concept of directionality, something Aristotle only rarely, yet significantly, takes into account. This will lead me to the idea that directionality represents a dispositional character of place, and only through this character can the exhaustivity of three dimensions be correctly justified.

Keywords: Aristotle; Three-dimensionality; Place; Directionality.

1. *Do we really need an argument for three-dimensionality?*

Aristotle's theory of space is a difficult matter in Aristotelian studies. This is perhaps because, as is often remarked, there is no properly Aristotelian theory of 'space'. Instead of speaking of space, Aristotle preferred to talk about 'place', 'spatial limit', 'figure', etc. But he made no statement (or, at least, only very few) in

favour of the idea of a space for itself¹. As M. Jammer notes, “since the notion of empty space is incompatible with his physics, Aristotle develops only a theory of positions in space, with the exclusion of the rejected conception of general space” (1993: 17). I cannot even attempt at a legitimation of this assertion. Nevertheless, I do think that Aristotle could help us clarify whether three-dimensionality is exhaustive or not. Before starting, it is worth making a few preliminary remarks.

First, one may legitimately ask whether we are entitled to think of a ‘justification’ of the necessity that objects and events in space are three-dimensional². There are two reasons to negate the entitlement. Historically, few attempts have been made to show that the number ‘three’ satisfies dimensionality of things in space. Aristotle does not make an exception to one such typically philosophical reluctance³. But there is a more important reason for us to feel the right to contrast a demonstration of the necessity of three-dimensionality. And this is eminently because every human being (and perhaps every being minded like us) can but experience things and facts of the world as three-dimensional. It is not just Aristotle’s, but Aristotle’s ‘*qua* human being’ theory of space that the latter be fulfilled by three dimensions⁴; for unless things and facts

¹ In what follows, I shall try to make Aristotle’s theory of dimensionality rest on no general account of space. I feel confident that room is left for so doing since Aristotle provides for no such account. Accordingly, no provisional definition of space is required in this paper.

² Of course, any philosophical justification of three-dimensionality is to be thought of in relation to ordinary experience. As H. Weyl remarks, the idea that there are three dimensions of space is a matter of contingency from a mathematical point of view. See Weyl 2009: 94. This is almost comprehensible, though, for there is ‘completeness’ in mathematics *per definitionem*, so for example, a one-dimensional line is said to be complete because a line is *per definitionem* a one-dimensional entity. Of course, physics too may help us clarify the nature of dimensionality, but I shall leave this issue aside in this paper.

³ Aristotle does not discuss the very concept of *dimension* in his work either. According to A. Falcon, “[c]i sono nozioni che sono ritenute così evidenti (o così fondamentali) da non essere mai spiegate facendo ricorso a qualcosa di più semplice. Per Aristotele la nozione di dimensione è una di queste” (2001: 29).

⁴ And this is because we all think that for something to exist is, in a broad sense, to be somewhere. “The view that everything which exists is somewhere is a com-

were in space, we would not be able to perceive or experience them. According to Aristotle, the body is (admittedly partly) defined as “what is extended in three-dimensions” (cf. P 209a4f.). Or, as he slightly differently, yet significantly, says elsewhere, it is “what is extended in all dimensions” (cf. P 204b21f.; compare OH 268a24f.)⁵. It is not just the case that we are psychologically inclined to conceive space and objects therein thanks to their three-dimensionality. Even in contemporary metaphysical debate it has been argued to interpret space as an essential component of the conceptual scheme underlying the possibility that we form a coherent account of experience. As P. F. Strawson has reasonably noticed, we can of course imagine a world in which no strictly spatial concepts occur. However, this should not be taken to mean that we do so without *any* concept of space, for such a world, if any, would be bound to include analogical correspondences to a concept of space (what Strawson labels ‘quasi-space’). Accordingly, a certain concept of space is thought of to belong necessarily to the very ordinary ‘reference-description’ practices implied in our experience⁶. Both reasons mentioned, however, eventually hinge on the Aristotelian idea of a philosophy brought into line with a common-sense view of the world⁷.

Even though there are good philosophical reasons not to attempt at a conceptual deduction of three-dimensionality, Aristotle

mon-sense view, metaphysically rather interesting, but also vague, because it is not clear what ‘somewhere’ really means” (B. Morison 2002: 19).

⁵ This evidently implies that no definition of three-dimensionality could ever be reached by deploying the concept of body. Were that the case, we would unavoidably run into a *circulus in definiendo*.

⁶ See his metaphysical essay on *Individuals*, especially the chapter on *Sounds*. Of course, Strawson’s definition of body is far removed from Aristotle’s, since the former takes ‘body’ to be what all our discourses ultimately refer to in order for them to be taken as conveying information about the world; whereas the latter almost defines the body as a three-dimensional entity inclined to movement. But I think it is worth mentioning his account so as to make sense of the (verily philosophical) belief that we can but take for granted that there is a space.

⁷ Against, namely, Plato’s *hyperphysical* theory. To oversimplify things, it is assumed that Aristotle represents the founding father of ‘realism’ in philosophy. This seems particularly evident from the *Metaphysics*; see e.g. M 99033f.

takes up the job explicitly in *On Heaven*⁸. But the argumentative efficacy of the proofs he provides is so weak that Aristotle's theory of place could not significantly benefit from them. Why then did he single out these proofs? Crudely put, Aristotle works out three distinct arguments. (a), he makes the first in light of the Pythagorean doctrine of the 'completeness' of the mathematical concept of number three (OH 268a10f.)⁹; (b), the second relies upon the idea that the number three has a performative function in the "worship of the Gods" (OH 268a13f.); (c), the last argues from the Greek grammar (OH 268a15f.) to the exhaustiveness of groups of (at least) three items (OH 268a15f.)¹⁰. It is clear that these arguments have several straightforward flaws. As a general objection, we should point out that, given that 'three' equates to 'all', we fail to see why there are more than three people on the earth¹¹. Additionally, it is easy to see that (a) falls back on a philosophical doctrine that Aristotle had no doubt was a dramatic failure¹². Meanwhile, (b) seems to fall short of philosophical relevance, primarily because of its standing or falling thanks to a contingent cultural set-up¹³. Finally, (c) implies that Aristotle could occasionally rely on linguistic phenomena so as to provide arguments in favour of an ontological thesis, which ultimately is historically doubtful at best

⁸ "Aristotle presents here the first argument in the history of Western philosophy for this conclusion" (G. Betegh/F. Pedriali/C. Pfeiffer 2013: 51), namely that physical bodies in space must be three-dimensional.

⁹ And this is because "the world and all that is in it is determined by the number three, since beginning and middle and end give the number of an 'all', and the number they give is the triad" (*Ibidem*).

¹⁰ Namely, that Greek speakers use 'all' to designate groups of at least three items. Aristotle outlines this argument a few lines earlier, see OH 268a7f.

¹¹ See C. Pfeiffer 2018: 77f.

¹² Certainly, Aristotle's most famous refutation of the Pythagorean ontology occurs in the *Metaphysics*. But he also significantly discusses the failure of such an account in *On Heaven* as he debates the constitution of natural bodies: "The same consequences follow from composing the heaven of numbers, as some of the Pythagoreans do who make all nature out of numbers. For natural bodies are manifestly endowed with weight and lightness, but as assemblage of units can neither be composed to form a body nor possess weight" (OH 300a14f.).

¹³ Aristotle argues this way in M 995a3f.: "The force of custom is shown by the laws, in whose case, with regard to the legendary and the childish elements in them, habit has more influence than our knowledge about them".

and conceptually untenable at worst¹⁴. But why did Aristotle think he was able to rule out the possibility that there were objects with more, or less, than three dimensions? How are we to understand the ‘suspect equivalence’¹⁵ he establishes between being three-dimensional and being complete? On what basis did he claim that

[a] magnitude if divisible one way is a line, if two ways a surface, and if three a body. Beyond these there is no other magnitude, and that which is divisible in three directions is divisible in all. (OH 268a9f.)¹⁶

My purpose here is to show that no argument in *On Heaven* can be conceived of as self-standing¹⁷. I will argue that concepts are involved that a text like *On Heaven* can no way explain (Section 2). I will then examine what function the concept of place has within Aristotle’s analysis of spatiality. For only through that concept can he explain in what sense the objects and events of the world are to be located in space. The concepts of limit and movement will also play a crucial role (Section 3). In the concluding paragraph, I will try to show that directionality is what ultimately grounds the possibility that there is a justification for the exhaustiveness of three-dimensionality. Roughly put, this is because three-dimensionality is not *primarily* a property of objects and events in space. On the contrary, it represents a *dispositional character* of the place itself (Section 4).

¹⁴ This way of arguing would indeed evidently infringe the ‘theoretical-priority-of-ontology’ thesis, though Ancient thinkers (and, among them, Aristotle in a very special manner) have often been met with the objection of having deduced the philosophically fundamental concepts from language. Admittedly, this could by no means fit the way Aristotle and other Greek philosophers thought of the theoretical foundation of such concepts, since the latter have an origin *in the thing itself*, as it were. Namely, they are ontological forms.

¹⁵ The locution is by S. Leggatt, see 1995: 170.

¹⁶ The word ‘direction’ does not occur textually. There are indeed even good reasons to replace ‘direction’ with ‘dimension’. See A. Falcon 2001: 28ft3. However, I do not take the issue to be significant to the discussion.

¹⁷ One way to answer is to say that Aristotle’s arguments take form from shareable premises. However, acceptance of this *ad hoc* solution would leave us with nothing more than the conviction that Aristotle should have been clearer that he actually was.

2. *Nature, principle and Aristotle's arguments in On Heaven*

As I have mentioned above, the arguments Aristotle provides on the very first page of *On Heaven* cannot serve his theoretical purpose. However, although he explicitly puts them into the text, I do believe he was well aware of their deficiencies. The first plausible signal of this is Aristotle's frequent deployment of the word 'nature'¹⁸. Indeed, in putting forth the 'Pythagorean' claim that the number three is perfect in nature, he does not merely argue from the mathematical concept of 'three', as Pythagoreans essentially do. On the contrary, he holds that there should be an *ontological grounding* of the perfection of the number three due to the very nature of "the world, and all that is in it". This is eminently because "beginning and middle and end give the number of an 'all'" (OH 268a11f.)¹⁹. Moving to the second argument, (b), we notice that fairly similar phrasing is used, for Aristotle claims that the completeness of the number three was derived "from nature (so to speak) as a law of it" (OH 268a13f.)²⁰. Nature, then, once again

¹⁸ It should be stressed that I do not take this first line of interpretation to stand for an argument (in the proper sense of the word). *Nature* is a polysemous concept which is liable to a variety of uses I shall not comment on in the paper. Cf. M 1014b16f. I have been told that this *polysemy* may undermine the present account. To answer this, I would be inclined to maintain that, being liberal about the meaning of nature we decide to pick out here, we can nonetheless suppose that Aristotle made use of this word so as to leave room for thinking that a strictly ontological analysis – as I see it, a kind of analysis that the books of *De Caelo* do not live up to – is required to prove three-dimensionality. However, this conjecture has the role both of a premise (which is to be justified due to the efficacy of interpretations) and of a guiding thread. Thanks to Prof. F. Aronadio and an anonymous reviewer for having pressed me to stress this.

¹⁹ Beginning, middle and end are by no means concepts one could derive from mathematics. Rather, they naturally belong to the ontologically grounding dimension. Certainly, the Pythagorean fundamental claim is that these two levels of foundation should not be distinct from one another. See G. Betegh/F. Pedriali/C. Pfeiffer 2013: 50. But this is not, and cannot be, Aristotle's account. At the end of the book, indeed, he will return to the inefficacy of any strictly mathematical depiction of the universe, cf. OH 300a14f.

²⁰ There seems to be an inferential procedure thus arguing from a non-cultural level, viz. nature, to religion as a significantly telling aspect of human experience.

seems to be presented as a priority character. Lastly, in the linguistic argument (c), Aristotle holds that, in using the quantifier ‘all’ for groups composed of at least three things, “we do but follow the lead which nature gives” (OH 268a20). What has been said so far suggests that we should not consider the arguments prompted in *On Heaven* as showing the exhaustivity of three dimensions. More than once, and perhaps strategically, Aristotle introduces a word, viz. ‘nature’, which we should refer to in the grounding level of ontology²¹. This will lead us to argue that the very three-dimensionality of bodies in space rather naturally falls back on ontological aspects that arguments in *On Heaven* do not take into account.

All of this brings us to the second and most telling signal of Aristotle’s awareness of the theoretical inefficacy of the proposed arguments. It pertains to the very status of a ‘science of nature’, that Aristotle claims is essentially concerned

with bodies and magnitudes and their properties and movements, but also with the principles of this sort of substance, as many as they may be. For of things constituted by nature some are bodies and magnitudes, some possess body and magnitude, and some are principles of things which possess these. (OH 268a2f.)

Three things are noteworthy here. First, there is a science thanks to which we become able to fully understand the general structure of bodies as they exist in nature²². Now, the concept of ‘nature’ collects all the constituents of this science under a uniquely determining label. That is to say, body, magnitude, property, movement and so on – all these concepts are *aspects* of nature²³.

²¹ Evidently, Aristotle’s concept of ontology is multifaceted. What I want to stress here is that once we realise that arguments from the *De Caelo* fail to provide a good reason to equate ‘three-dimensional’ with ‘perfect’, then we are quite reasonably led to a kind of analysis that we could term ‘ontological’ in the broad sense of the adjective, namely as one concerned to show the fundamental properties and dispositions of what there is. This is fairly in line with what I am about to argue.

²² On this issue, see G. Betegh/F. Pedriali/C. Pfeiffer 2013: 33f.

²³ I use the word ‘aspect’ in the most general sense, namely, as referring to the *descriptive power* that concepts falling under the general notion of ‘nature’ have. I think

Secondly, and closely related to the first point, the concept of 'nature' manifestly strictly relates to the concept 'substance'. This is primarily because all that is constituted by nature ultimately amounts to a *substantial form*. Aristotle discusses the connection between 'nature' and 'substance' densely in his *Metaphysics*, as he claims that

It is plain that nature in the primary and strict sense is the substance of things which have in themselves, as such, a source of movement; for the matter is called the nature because it is qualified to receive this and processes of becoming and growing are called nature because they are movements proceeding from this. And nature in this sense is the source of the movement of natural objects, being present in them somehow, either potentially or actually. (M 1015a13)

It is worth noting that Aristotle establishes a strict relation between nature as the substance of natural things on the one hand, and *movement* as an essential property of natural things on the other hand. All that remains and changes distinctively depends upon the fact that there is a nature that constitutes it as the very substance it is²⁴. It follows that the ground for things to have the source of their movement within them, essentially means that they are 'natural'. But for them to be what they are, it is also necessary for them to be located somewhere, i.e. in place. Now, as we shall see more

this is conveniently consistent with Aristotle's phrasing. Of course, 'nature' has an ontological value the word 'aspect' cannot be apt to. See e.g. P 192b8f. Interestingly enough, Aristotle there argues for natural science as concerned both with form and matter, since nature itself is also defined, among other things, in terms of form and matter (see P 192a12). In what follows, I shall try to show that Aristotle's theory of place is to be confined to analysing a *formal aspect* of bodily entities. Put simply, a preliminary ontological analysis is called for since, as Aristotle puts it, "it is the job of first philosophy" (P 194b17) to make sense of the *separability* of forms from matters.

²⁴ Cf. e.g. P 200b12f: "nature is a principle of change, so if we do not understand the process of change, we will not understand nature either". On the relation between nature and movement, see H. S. Lang 1998: 34f. It is noteworthy that, in the lines that immediately follow the quoted passage, Aristotle speaks of "what is common" to change as such and mentions, among other things, the very concept of place. Place is thus revealed to be a general property belonging to any kind of change that may possibly occur.

clearly in later steps of the reconstruction, being mobile and being in a place are two fundamental properties of bodies as a topic of natural science. Indeed, this is what differentiates them from bare geometrical solids²⁵.

There is a third thing that should be noted: Aristotle explicitly says that there are ‘principles’ of things taken as natural bodies. As he makes clear in the *Metaphysics*, the concept of ‘principle’ contains a variety of meanings, and one of them is highly telling for the theoretical purpose of the paper. According to the fourth meaning identified by Aristotle, a principle consists in

That from which (*not* as an immanent part) a thing first arises, and from which the movement or the change naturally first proceeds. (M 1013a7f.)

Roughly put, the principle of a thing is the ‘cause’ of its having a property or a capacity, such as those of moving and changing. Certainly, the idea of *causality* brings in an ontological value not all principles properly fulfil. But nothing prevents us from saying that ‘cause’ may be equally legitimately conceived in terms of a ‘conceptual condition’ allowing us to justify the thing’s owning a property or a capacity. Aristotle argues this way, claiming that a principle is “that from which a thing can first be known” (M 1013a14f.)²⁶. It is fairly true that these two meanings of the same concept should not be confused with one another. But, from an Aristotelian point of view, they are meant to be genuinely interrelated²⁷.

What has been said so far, then, seems to attest that a much more fundamental kind of analysis is called for if we are to correctly understand the essence of three-dimensionality. This is mainly because there is a naturally established principle underdetermining

²⁵ See A. Falcon 2005: 38f.

²⁶ Compare Aristotle’s discussion of the principle of non-contradiction at M 1005b5f. Certainly the principle of non-contradiction is logically the most fundamental rule of syllogistic reasoning. On the other hand, it represents the ontologically primary principle of all being.

²⁷ Cf. M 1013a17f.: “It is common, then, to all to be the first point from which a thing either is or comes to be or is known”.

all bodies – a principle granting that it cannot be the case for a body to have more than three dimensions. Certainly, a merely empirical description of how objects and events in the world are constituted appears to fall short of explanation. The study of being, i.e. of the fundamental properties and dispositions of what there is, is thus mandatory to this effect. However, even physical investigation for Aristotle is designed to improve, perhaps noticeably, our understanding of how the three-dimensionality of space should be taken into account²⁸.

3. *Place, dimension and limit. Aristotle's 'Transitive Containment Theory'*

Thanks to Aristotle's tacit admonitions, we are then led to his *Physics*. This represents a brilliantly pursued, admirable work in natural philosophy, the historico-philosophical echoes of which are hard to underestimate²⁹. There is one chapter of the book that can greatly aid us in explaining Aristotle's confused standpoint on the nature of three-dimensionality. It is the chapter dealing with the concept of 'place'. According to Aristotle, place denotes "the limit of the containing body" (P 212a5). This means that it does not at least primarily commit to what is contained in the containing body³⁰. Nor does the concept of place, qua limit of the con-

²⁸ Of course, Aristotle's conception of physics should not be confused with modern one, because the former is thought to have an ontological value the latter does not.

²⁹ See M. Jammer 1993.

³⁰ Aristotle seems reluctant to identify the place with the limit of the object, for this would make place the form (viz. shape) of that object. In another respect, the place could be equally consistently conceived as the matter of the object, for it "also seems to be the extension of the magnitude of a thing, and taken in this way it is the thing's matter" (P 209b6f.). But such an account would dramatically lead to Plato's identification between matter and space. One can then see Aristotle arguing for a *multi-level relational* conception of place – the theory according to which every body being placed somewhere must in turn be conceived as 'limited' by a containing body. The coincidence of the limits between the *containing* and the *contained* constitutes the place. This explains Aristotle's belief that "[a] body is in place, then, if there is a body outside it which contains it, but not if there is none" (P 212a31f.). See J. Mariña 1993: 212f. This marginally sheds

taining body, commit to a specified body of which it is supposed to identify the limit. For Aristotle's account of place seems to underline that there is a conceptual independence between place and body, so we can conceive of a definition of the former as barely nominally linked with the concept of the latter³¹. Accordingly, Aristotle makes use of the phenomenon of replacement to show that place does not depend notionally on the body occupying it³². In line with what we have already seen, this means that place is a *non-immanent principle* of the possibility to locate bodies in space³³. Surely, this is not to say *eo ipso* that place reflects an ontological ground for bodies as such. But for a body to be what it is, it seems necessary for it to be located 'somewhere'. This, Aristotle argues, entails that we are endowed with a concept of place that does not depend on any specified concept of body. For if there were no place, there would be no body qua occupant of space. Aristotle's theory of place thus results in a 'Transitive Containment Theory' (TCT), stating that objects and events are in space *if and only if* there are places ready to be occupied³⁴.

light on the thesis that the world as a whole is not in a place (cf. P 212b10f.) so a *regressus ad infinitum* is easily inhibited. Further readings are also possible, which I shall not discuss here. See e.g. B. Morison 2002: 137f. & D. Bostock 2006: 128f.

- ³¹ Whether or not this conceptual independence brings in the ontological dependence of body on place is not at stake here. Thanks to an anonymous reviewer for having urged me to underline this point.
- ³² Aristotle absolutely holds the thesis that place "can be left behind by the object and is separable from it" (P 210b8f.). Admittedly, we appear to be able to mention, or refer to, a place regardless of it being occupied by a body. See P 208b1f. Cf. B. Morison's discussion of the topic (2002: 21f.). See also D. Quarantotto 2017: 61f.: "Lo scambio di posto è particolarmente adatto a suggerire l'esistenza del luogo perché, chiamando in causa due corpi e presentandoli come enti uno dei quali prende il posto dell'altro, evidenzia che, oltre ai due corpi che si scambiano di posto, *esiste qualcos'altro*, che è condiviso da entrambi (nel senso che è ciò che essi si scambiano) e *diverso* da entrambi" (here p. 62).
- ³³ As B. Morison puts it, "[p]laces have power, i.e. causal significance" (2002: 52). However, this should not be read as though places were causes *stricto sensu*. Perhaps the best way to conceive of it would be through the concept of 'that in virtue of which', as is explained at M 1022a14f.
- ³⁴ This brings in a difficulty I shall only marginally discuss, namely, the fact that Aristotle thinks of place at once as separable and inseparable from the body. For a classic discussion of the topic, see R. Sorabji 1988: 188f. Certainly, the occupier

Accordingly, properties and capacities one would intuitively attach to objects and events in space are to be conceived, for Aristotle, as properties and capacities of places those objects and events (perhaps contingently) occupy. And one of them, quite surprisingly, is the three-dimensionality. According to Aristotle's view, every place

is three-dimensional; it does have length, breadth, and depth, by which every body is defined. (P 209a4f.)

It is not true then, as every human being would be inclined to maintain, that bodies themselves are three-dimensional. According to the Aristotelian TCT, three-dimensionality would persist in existence even if there were no object at all³⁵. It follows that the definitional relation, so to speak, proceeds from the nature of place to that of body: what defines place defines the body in turn. But how are we to make sense of this idea (departing from common-sense views) that three-dimensionality does not belong to bodies? Does Aristotle provide arguments in support of this philosophically radical thesis? I suggest that we can make sense of the thesis only if

of a place is not necessary for that place to be conceived (besides, one can argue that we could make use of indexicals such as 'here' or 'there' without being committed to any specified body). Still, Aristotle makes it clear that place consists in the limit of the containing body, which essentially is a kind of body. What is more, one can easily argue from this that such a limiting body is contained in another body, for contact between bodies is essential to define place. See D. Quarantotto 2017: 173. This would make the same body both a *containing* and a *contained*, though in different respects (that is, not contradictorily). As many scholars have remarked, Aristotle's theory of place ultimately commits to the idea that place and body are inseparable. Cf. for instance H. Mendell 1987; D. Sedley 2012; E. Mamchur 2016. I am only partly in agreement with these authors. This is because they fail to take account of Aristotle's belief that there are properties and capacities of bodies in space that do not depend on body at all. These are, for example, three-dimensionality and directionality. I shall return to this later in the paper.

³⁵ Advocates of TCT are thus naturally forced to admit that three-dimensionality is what ultimately *contains* bodies insofar as they are located somewhere. Cf. H. S. Lang 1998: 110: "Place is three-dimensional not as another body determined to be three-dimensional, but as a principle, a limit, that determines the cosmos in respect to 'where'".

we are able to dig Aristotle's argument out of his texts. And, I will argue, this shall once again lead us to the *Metaphysics*.

I hold that the very central concept of Aristotle's reasoning is that of 'limit'. Admittedly, Aristotle does not expand upon this concept in the *Physics*. This is perhaps because 'limit' is in essence a metaphysical concept a barely physical investigation cannot take into account exhaustively enough. In the *Metaphysics*, indeed, Aristotle identifies four different meanings of the concept of limit. I will primarily focus on the first meaning Aristotle singles out. Here, the limit is taken to be

the last point of each thing, i.e. the first point beyond which it is not possible to find any part, and the first point within which every part is.
(M 1222a4f.)

Allegedly, Aristotle argues for a 'spatial' conception of limit, which fittingly squares with his conviction that only thanks to place can one think of spatiality in the right way. But, as has been said, place is logically based upon the idea of limit. This further matches the fourth and last meaning of the concept of limit – the one stating that 'limit' applies "to the substance of each thing, and the essence of each" (M 1022a8f.). In light of this, one can then well maintain that 'limit' represents the metaphysical concept granting the possibility for something to be what it is. But this in turn implies that 'spatiality', insofar as it is based upon the metaphysical concept of 'limit', sets forth a key ontological concept, for no-one could ever conceive of a body that is not in space (that is, located in a place). By the same token, one can easily conclude that three-dimensionality no longer represents an accidental property or capacity of objects and events located in space. On the contrary, it attaches to the very essence of all there is, to the extent that it is somewhere³⁶. But, one might further ask, how are we to deduce

³⁶ Cf. also C. Pfeiffer 2018: 88f., though he believes that three-dimensionality issues from considering physical bodies rather than place, as I do. Certainly, *for us* to know actual places, there must be occupiers of space. This is because, if there were no concrete extension borne by things, there would in turn be no delimitation in space, and thereby no place. Nevertheless, a limit is conceptually independent of what is limited, at least as long as we can provide for a very abstract

the *exhaustiveness* of three dimensions in space? Could not we think of other, even simpler, spatial dispositions so that there are two-dimensional things to be experienced? A merely factual proof does not seem satisfactory to this effect – thus ontological analysis is required once more.

4. *Directionality and Aristotle's 'good argument' for three-dimensionality*

According to Aristotle, there is no direct inference from being a body to being located in space (and, thereby, being three-dimensional). As in the case of a body's being in a place only to the extent that it is 'contiguous' with another body – a body is located in a place, namely it can be said to be 'somewhere', if and only if it is subject to *movement*³⁷.

The first point to appreciate is that it would never occur to us to make place a topic for investigation if there were no such thing as change of place. (P 211a12f.)

Furthermore, place is said to be the "limit of the containing body" only to the extent that "what it contains" consists in "a body which is capable of movement" (P 212a6). It is not just the fact that no investigation into place and space would ever be carried out without there being a change of place. What I think Aristotle wants to say is that we would not be able to *conceive of* place

definition of limit that contains no reference to a specified 'limited'. It is not, as it were, a matter of existence, but of what we can understand as bearing a conceptual priority that Aristotle seems concerned with.

³⁷ This means that the transitivity of Aristotle's theory of 'placement' involves a third moment, that of the movability of body in space. That is, a body is somewhere if and only if (a) there is a place as the limit of its containing body, but (a) can be satisfied if and only if (b) the body under discussion is a movable entity. 'Inclination to movement' is important here for allows us to differentiate mathematical from physical bodies. See A. Falcon 2001: 47f.

and space unless there were a kind of change, such as movement³⁸. ‘Change of place’ thus represents a ‘principle’ in the sense already discussed, namely as a conceptual condition of the possibility that there is something like place and space. Aristotle also refers to ‘change of place’ in terms of ‘variation’ (cf. P 225b10)³⁹. And this variation – movement, in general – essentially indicates the very first kind of change⁴⁰. But, then, what does the variation under discussion consist in? If the concept of body qua occupant of place were required, then Aristotle’s overall account of place would fail to be consistent. In fact, Aristotle thought of place as non-trivially separable from the body possibly occupying it. A good way of getting out of this awkwardness is to make use of the concept of *directionality*⁴¹.

In a text from the *Physics*, Aristotle claims that place and direction are two strictly related concepts. One such connection is stated on account of Aristotle’s belief that place helps us to correctly explain the capacity of body to be spatially moving. I think it is worth quoting the text in full:

the movements of the simple natural bodies (fire, earth, and so on) show not only that there is such a thing as place, but also that it has a certain power. For unless prevented from doing so, each of them moves

³⁸ As J. Mariña correctly observes, “in order to have a notion of place as something knowable independently from what occupies it, we must first possess a notion of substantial entities which change place” (1993: 207).

³⁹ According to Aristotle, such a variation consists in a change from an entity to an entity (and, therefore, does not involve any non-entity); cf. P 224b35f.

⁴⁰ A justification of this assertion cannot be fully pursued here, for it presupposes Aristotle’s concepts of the ‘first agent of change’ and of ‘continuity’, as is patently shown at P 260b22f. On Aristotle’s conception of agency in relation to movement, see A. Marmodoro 2018: 21f. On continuity of place, cf. C 5a6f.; but see also S. Waterlow 1982: 149f.; and D. Quarantotto 2017: 165f. The problematic relation between the theory of place in C and the one in P has been analysed by H. Mendell 1983.

⁴¹ Of course, movement cannot be exhaustively conceived without referring to the concept of a substantial entity, viz. a body that moves. But I think Aristotle was aware of the difficulty he would have faced once he had admitted that ‘movement’ under study essentially involves the concept of body. I therefore believe he had room for making claims of a different kind, and this thanks to the concept of ‘directionality’.

to its own place, which may be either above or below where it was. Above and below and the other four directions are the parts or forms of place. Directions like above, below, right, and left are not just relative to us. In the sense in which they are relative to us, they are not always the same, but depend on our position—that is, on which way we are facing. That is why the same object might well be, at different times, to our right and to our left, above us and below us, in front of us and behind us. But in themselves each of the six directions is distinct and separate. (P 208b8f.)

There are several points that should be noted here. First, Aristotle takes it that 'place' as the condition for things to be in space has a 'power' upon those things. This is prominently because it determines the possibility for things to move *directionally*⁴². Secondly, place is said to be composed of 'parts or forms', each of which basically consists in a direction⁴³. Thirdly, directionality conceived as a constituent of place itself is not to be taken as conceptually dependent upon our having a perception of it. That we could determine a direction of a concretely moving body as, say, 'the above' does not affect the objective way in which that body actually moves. It follows, fourthly, that directionality remains independently of its being centred on the subject organising her worldly experience. But, one might ask, what does this conceptual independence ultimately imply? That directions do not adhere to the perceptual subject seems to entail *either* that they are a property or a capacity of moving bodies, *or* that they derive from the very ontological nature of place as discussed so far. I strongly believe that several counter-arguments may be adduced against the thesis that

⁴² See B. Morison 2002: 36f.

⁴³ Compare P 205b31f: "Besides, every perceptible body occupies a place, and the forms and varieties of place are above, below, forward, back, right, and left. These are not mere human conventions, but objective divisions of the universe". See the corresponding note of the editor at p. 250f. R. Sorabji offers a completely different account, according to which Aristotle would have presented "a thing's place as a two-dimensional surrounding surface, rather than as a three-dimensional extension or interval" (1988: 187). However, I do not see how he could maintain this.

directionality belongs to the movables in space as if it were a property or capacity of them.

Roughly put, the thesis that directionality appertains to bodies as a property or capacity naturally implies that no direction could exist independently of a body's being able to follow it⁴⁴. There are, I think, several objections to this reading. Firstly, if the body were the ultimate condition for directionality to be explainable, then we would be obliged to say, in light of what Aristotle himself claims, that place is composed of 'parts or forms' that do not belong to place itself, for they instead come to be in virtue of the movable body. But this conclusion hardly seems to match Aristotle's⁴⁵. Secondly, by stating a dependence of direction upon body, it would be fairly easy to maintain that a given place was 'created' or 'engendered' only thanks to the body effectively moving to that place. Aristotle seems far removed from similar doctrine⁴⁶. Thirdly, if directionality were dependent on the existence of bodies, the possibility that there was a 'merely subjective' appraisal of directions would make no sense at all. For only through the perception of a

⁴⁴ For example if there were two places, I could not say that one was behind or to the right of the other.

⁴⁵ See the list of properties belonging to place at P 209a2f.

⁴⁶ D. Sedley (2012) and J. Lennox (2009) suffer from the same failing assumption that the nature of body could significantly contribute to the description of the essence of place. The former claims that Aristotle's theory of place should be replaced with a theory of 'natural places', thereby being committed to the fundamental function of a group of elemental bodies. Criticism to this approach was raised by B. Morison 2002: 34f.; 50f. I think Aristotle's theory of natural place can play no role within his theory of directionality for at least two reasons. The first is that, as suggested by B. Morison, the very syntagma 'natural place' is not Aristotle's. If we are to attach the adjective 'natural' to a place, then we can do so only *transitively*, i.e. by assuming that there are elemental bodies that are supposed to naturally occupy, or tend to occupy, such place. This brings in a second reason I shall mention cursorily, namely the idea that the way I am here talking about 'place' differs radically from the way a theory of natural place would. In fact, if we accepted a definition of place as based on the theory of natural place, we would be content to argue that a place is defined in terms of the body that naturally occupies it, whereas I contend that there could be no definition of place if it committed to specified kinds of bodily entities, though they may be informative about what there is. Analogously, we can argue against J. Lennox, who instead maintains that dimensionality, given that it is grounded in direction, basically issues from animal kinds.

body moving in space would one be entitled to say there is something like a direction⁴⁷. Yet Aristotle expressly makes room for a merely subjective appraisal of directions⁴⁸. Fourthly, and related to the previous point, were it the case that the body makes directionality possible, then it would in turn be licit to say that Aristotle's theory of space lacks any 'relational' character, and that it would boil down to a form of 'monadic' space. This is essentially because each body would convey a definite conception of space as dependent upon being movable *itself*: Aristotle's account of space would so be committed to the thesis that there are as many spaces as bodies moving directionally⁴⁹. To my mind, the only way of making Aristotle's theory of directionality a consistent one is to say that directionality represents a property or capacity of place itself.

Aristotle's conception of directionality is far from being easy to understand. Perhaps he put together concepts that he should have kept distinct. I would like to return to where we left off at the beginning of the paper, namely *On Heaven*. There Aristotle speaks of directions as essential constituents of the celestial bodies, though he does so in a very general fashion.

The distinctions are three, namely, above and below, front and its opposite, right and left—all these three oppositions we expect to find in perfect body—and each may be called a principle. Above is the principle of length, right of breadth, front of depth. (OH 284b24f.)

What I think Aristotle is doing here is setting out a class of distinctions by means of which he becomes able to distinguish 'directions' in space. The reference to the 'perfect body' is not surpris-

⁴⁷ And this should naturally comply with the very directional movement of the body, otherwise it would be nothing at all.

⁴⁸ Cf. OH 285a1f.: "It is true that we speak of above and below, right and left, in these bodies relatively to ourselves. The reference may be to our own right hands, as with the diviner, or to some similarity to our own members, such as the parts of a statue possess; or we may take the contrary spatial order, calling right that which is to our left, and left that which is to our right".

⁴⁹ I think this is fundamentally the very upshot of E. Machmur's interpretation. Relationality of Aristotle's space can be explained even in terms of a place-dependence of directionality.

ing, for it blatantly reminds us of Aristotle's thinking of three-dimensionality as a mark of completeness of what there is⁵⁰. A given body is 'perfect' insofar as it is conceived in terms of length, breadth and depth. But to be conceived this way means that a perfect body is the body that can move either above or below, right or left, front or behind⁵¹. And even though movements of this kind may be legitimately understood in mathematical terms, Aristotle does not want to make any mathematical (or mathematising) claim⁵². The three couples involved in directionality thus basically put forward the very primitive manner in which objects and events in space are arranged. Unsurprisingly then, Aristotle claims that the 'above' (and the 'below'), the 'right' (and the 'left'), the 'front' (and the 'behind') are all 'principle'⁵³ in a very fundamental sense. But it should be noted once more that they do not belong to the bodily entity. For something can move right or left only if the relevant directions are presupposed. In this way, directions are a non-immanent principle of groups of translations through space.

In light of the above, even the 'relational' character of space typically associated with Aristotle's theory of place, find adequate explanation. It is not just that things could be individuated in a portion of space only to the extent that they are in relation to one another. More radically, I think Aristotle is saying that no place could ever be conceived at all, unless it was in relation to other potentially infinite places. And all of them stand in a *directional relation*⁵⁴. As we have seen already, the directionality embracing locations of space does not fall back on the bodies possibly occupying such locations. This means that the directions in space could be

⁵⁰ Of course, bodies are perfect only insofar as we conceive of them as three-dimensional, and not as parts of the universe. See A. Falcon 2001: 33.

⁵¹ In this sense, Aristotle claims that "anything that is in a place is so because it is somewhere, and that is to say that it is above or below or in one of the other six directions" (P 206a3f.).

⁵² I do believe, along other commentators, that no mathematical deduction of three-dimensionality lives up to Aristotle's definition of three-dimensional physical bodies. See G. Betegh/F. Pedriali/C. Pfeiffer 2013: esp. ft. 18.

⁵³ The parenthetical integrations are justified on the basis of Aristotle's text. On Aristotle's 'axiology' of direction, see J. Lennox 2009: 191f.

⁵⁴ Namely, in a relation based on the possibility to determine whether two things are, say, one on the right side of the other, and so for all other directions.

thought of fully even if there were no body at all⁵⁵. From an Aristotelian point of view, this in turn entails that they *exist* independently of the actual being of bodies. Thanks to this both conceptual and ontological independence, directionality can be quite fittingly understood in terms of *dispositional character* of the place⁵⁶. Every place has directionality as a stable property which in turn involves that it relates to all other places in some definite and specified manner. By the same token, the directionality of place basically ontologically grounds the fact that there can be no body that exceeds 'three' dimensions. This analysis hence shows that Aristotle had a good argument for proving the exhaustiveness of the three-dimensionality of objects in space, and that one such proof is to be based upon his theory of directionality in the form of dispositional character of place.

Admittedly, Aristotle's argument for three-dimensionality cannot be completely understood unless other concepts are involved. And things immediately become more difficult, for instance, as soon as we consider Aristotle's idea that 'where' and 'being in' are categories⁵⁷. By the same token, we should also face what C. Pfeiffer suggests is Aristotle's theory of a 'quasi-generation' of (three-dimensional) physical bodies, namely the theory that bodies are formed progressively from lower-dimensional (i.e. one- or two-dimensional) levels up to three-dimensionality⁵⁸. No doubt this should be done if we are to gain a full view on Aristotle's account of space and spatial occupant. But this is beyond the scope of this paper, which simply shows that Aristotle's argument in favour of three-dimensionality does not rely on ill-stated assumptions, but is

⁵⁵ It should be noted that I do not argue that directions, as well as places, would *exist* without bodies being able to move directionally. Once more, what I have been arguing is that directions, as well as places, are notionally independent of bodies. Accordingly, we can think of directionality as essentially constituting the *concept of* bodily entity. However, a tension remains in Aristotle as to whether conceptual independence logically implies ontological priority, but I cannot discuss this topic further in this paper.

⁵⁶ By 'dispositional character' I would like to denote the 'having' at M 1022b4f., which basically indicates a 'disposition' had *in a stable manner*.

⁵⁷ This reference is suggested by a reading of P 201a3f.

⁵⁸ See C. Pfeiffer 2018: 85.

due to a systematic account of what it means for something to be located somewhere⁵⁹.

⁵⁹ I have been charged with the objection that little improvement is achieved once we hold that Aristotle's equivalence between 'three-dimensional' and 'perfect' should be grounded in the concept of directionality, rather than in that of dimensionality. One might reasonably ask whether we have reason to affirm that no more than three (couples of) directions could possibly occur in a theory. What ensures that we should not provide for a further argument for the exhaustivity of 'three-directionality'? I would contrast this objection by just saying that no apodictic argument has been sought for in this paper. My main concern was to show that Aristotle's argument for three-dimensionality, if any, is not based on ill-stated assumptions and mostly coheres with his general philosophical framework. Perhaps it is no matter of philosophy to prove apodictically that objects in space cannot have more than three dimensions. It is however up to philosophy to *chart a course* so as to make the possibility of a proof intelligible.

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Abbreviations of Aristotle's works

- (OH) *On Heaven*, (1984), *The Complete Works of Aristotle. The Revised Oxford Translation*, I, J. Barnes (ed.), Princeton University Press: Princeton; 447-511.
- (M) *Metaphysics*, (1984), *The Complete Works of Aristotle. The Revised Oxford Translation*, II, J. Barnes (ed.), Princeton University Press: Princeton; 1552-1728.
- (P) *Physics*, (1996), *Oxford World's Classics*, Waterfield R. (trans.), Oxford University Press: Oxford.
- (C) *Categories*, (1984), *The Complete Works of Aristotle. The Revised Oxford Translation*, I, J. Barnes (ed.), Princeton University Press: Princeton; 3-24.

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